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A Mathematical Model to Break the Life Cycle of Anopheles Mosquito.

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Abstract:

A mathematical model to break the life cycle of anopheles mosquito Is derived, aimed at controlling or eradicating mosquito by introduction of natural enemy "copepods" (an organism that eats up mosquito at larva stage). Thereby reducing the menace of malaria parasite in our societies. The model equations are derived using the model parameters. The stability of the free equilibrium states is analyzed using the idea of Beltrami and Diekmann. From the stability analysis we observe that $R_O < 1$ which implies that the model free equilibrium state is locally asymptotically stable. Hence the number of larva that would transform to pupa is almost zero and that means the life cycle can be broken at the larva stage with the introduction of natural enemy.

Keywords: Mathematical model, Anopheles Mosquito, Equilibrium states, Reproduction number.

Introduction:

Every year, one to three million deaths is attributed to malaria parasite in sub-Saharan Africa and in Nigeria in particular out of which one third are children. Malaria is transmitted by the female Anopheles mosquito which feeds on human blood. Much work has been done to genetically modify mosquitoes in the laboratory to hinder or block parasite transmission thus, making the mosquitoes refractory. This is done by insertion of genes at appropriate site to create stable germline. The progress in this area is fairly recent. A review of transgenic mosquitoes for suppressing the transmission of malaria is presented by (Christophids 1985). However in 1967, World Health Organization (WHO) realized that the global eradication of malaria was impossible for a variety of reasons and the control focus shifted to the control of deadly diseases. Since the idea of eradicating mosquitoes was not realistic, the efforts were directed towards the reduction and management of their population below the threshold that would cause diseases. More also, for the first time in Africa an entomological study went beyond the conventional practice of determining parity and survival rates of field collected adult Anopheles mosquitoes but, also related these variables to duration of Plasmodium sporogeny and estimated the expectation of infective life. Hence bloods seeking female mosquitoes were collected in Ilorin, Nigeria, from January2005 to December, 2006. The Anopheles gambiae population in Ilorin is dominated by older mosquitoes with high survival rate thereby, suggesting a high vector potential for the species in the area. This information on the survival ratio of Anopheles gambiae in relation to malaria transmission would enhance the development of a more focused and informed vector control interventions.

In 1963, WHO team also carried out an extended field trial with dichlrvos in Kankiya district of the then Katsina province, Northern Nigeria (Foll et al 1965), the study has been made of the conditions of malaria transmission. It was found that malaria transmission occurs principally from August to December but continues at much reduced scale through out the rest of the year even when Anopheles densities are as low as 0.02 per hut.

Malaria remains a major killer with more than one million deaths each year in sub-Saharan Africa. In Nigeria, malaria is responsible for about 30,000 deaths every year and accounts for 40% public health expenditure. The cost of malaria treatment and prevention in Nigeria has been estimated to be over $1 billion per annum.

However with a project work such as mathematical model to break the life cycle of anopheles mosquito which is the major carrier of plasmodium that transmit malaria will reduce or eradicate the risk of malaria parasite. Hence the money spent on the burden of malaria by Nigerian Government and WHO's "Roll Back Malaria" programs will be concentrated on something else.

Methodology:

It is true that anopheles mosquito play a critical role in the life cycle of plasmodium. Adult female mosquito which is Anopheles mosquito is the major medium of transmitting malaria infection to man. Mosquito (anopheles) complete a generation during the delay in humans and as a consequence, the proportion of mosquitoes that is infectious changes rapidly in response to changes in the proportion of humans that are infectious. In this paper we propose a mathematical model to break the life cycle of anopheles mosquito by introducing the natural enemy (copepods).The population is partitioned into four compartments of; Adult A (t), Egg B (t), Larva L (t), Pupa P (t).

In this model we will consider what happens at each stage, we shall also represent each compartment by a differential equation.

Basic Definitions of Model Parameters:

A (t) = number of adult mosquito at time (t)

B (t) = number of egg at time (t)

L (t) = number of larvae at time (t)

P (t) = number of pupa at time (t)

ŋ = the incidence rate (the rate at which adult mosquitoes oviposit)

b = natural birthrate

QUOTE μ μ = natural death

N (t) = total population

$S(t)$ =natural enemy

QUOTE σ σ = the proportion at which egg harsh to larva

λ = the proportion of larva that harsh to pupa

QUOTE ρ ρ = the proportion of pupa that develop to adult

QUOTE α α = the rate of natural enemy that eat up the mosquito larva at time (t)

c = the average temperature of water culture

Pictorial Representation of the Model:

β =probability of larva been eaten up by larva

Assumptions of the Model:

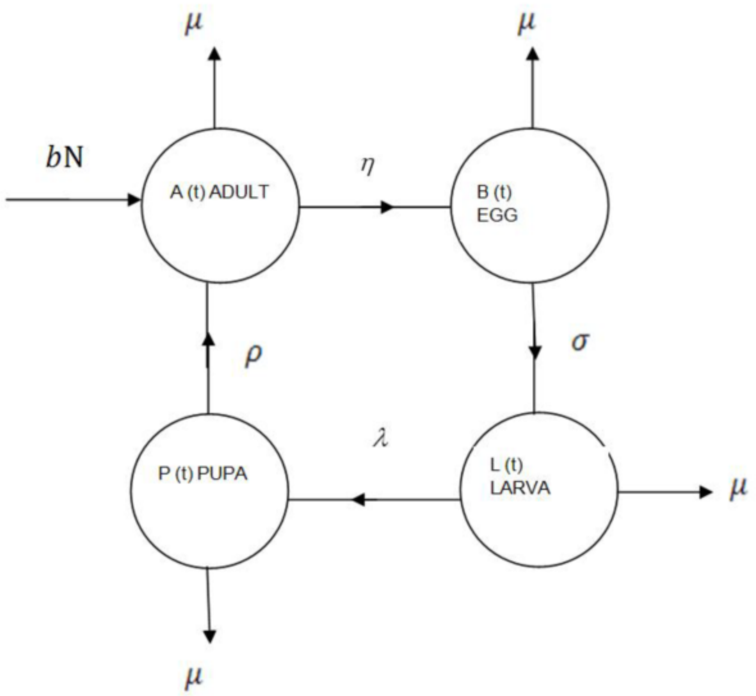
a) The total population of anopheles mosquito consists of four sub-populations.

b) Mosquitoes are given birth to at a rate (b) where the eggs are layed and they die at a rate QUOTE $(\mu$ $(\mu$) by introducing a natural enemy $S(t)$ to the larva stage at the rate QUOTE $\alpha(t)$ $\alpha(t)$

c) The parasite of one mosquito transferred from one mosquito to the other only through the medium of host i.e. through horizontal transmission.

d) Emigration and immigration do not occur in this population; however the population increases only through natural birth rate and decreases only through natural death rate.

e) Anopheles mosquito is assumed to transmit malaria only through direct contact.



Figure(1): The flow diagram

The Model Equations:

$$N(t) = A(t) + B(t) + L(t) + P(t)$$

$$\frac{dA}{dt} = A'(t) = bN - \eta A(t) - \mu A(t) + \rho P(t) \qquad \dots \quad \dots \quad \dots (3.5.1)$$

$$= bN + \rho P(t) - (\eta + \mu) A(t)$$

$$\frac{dB}{dt} = B'(t) = \eta A(t) - \sigma B(t) - \mu B(t) \qquad \dots \quad \dots \quad \dots (3.5.2)$$

$$= \eta A(t) - (\sigma + \mu) B(t)$$

$$\frac{dL}{dt} = L'(t) = \sigma B(t) + \alpha(t) S(t) - \lambda L(t) - \mu L(t) \qquad \dots \quad \dots \quad \dots (3.5.3)$$

$$= \sigma B(t) + \alpha(t) S(t) - (\lambda + \mu) L(t)$$

$$\frac{dP}{dt} = P'(t) = \lambda L(t) - \rho P(t) - \mu P(t) \qquad \dots \quad \dots \quad \dots (3.5.4)$$

$$= \lambda L(t) - (\rho + \mu) P(t)$$

Natural Enemy

In this work we proposed breaking the life cycle of anopheles mosquito using natural enemy which is a biological control method for mosquitoes, thus, reducing or eliminating the menace of mosquito. We will explain briefly the two forms of natural enemies to be used for the success of this work.

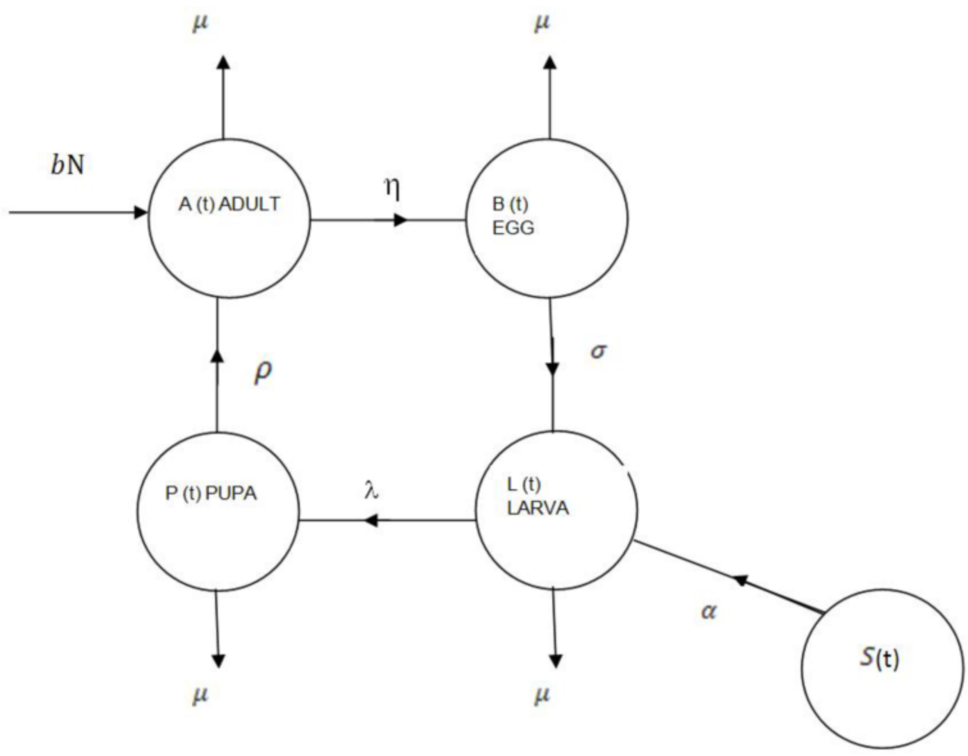
Copepods

Copepods are tiny crustaceans (shrimps, crabs, lobster and relatives) that are wide spread in both fresh and salt water habitat. They are voracious predators used to control mosquito production from water holding areas. For effective mosquito control with copepods, knowing where the mosquitoes breed is very essential. A single copepod, consumes approximately 30 first instars of the mosquito larvae per day or more.

Predatory mosquitoes

These are the largest mosquitoes (Diptera culicidae) in the world and are members of the genus Toxorhynchites. They are found primarily in the tropical forested areas. This generic name replaced megarhinus which is pre occupied (stone 1948). Mosquitoes in the genus Toxorhynchites are predaceous as larvae on the immature stages of other mosquito species and often turn cannibalistic. They feed on nectar and other natural occurring carbohydrate sources but never take blood meals, this lead many entomologist to recognize the mosquito's potential to reduce pest and disease bearing mosquitoes. This predatory mosquitoes are the most common arthropods that have been used for control of "container breeding" mosquitoes. One of the advantages of a Toxorhynchites is that the adult mosquito can disperse and lay eggs in area most likely to escape treatment with insecticides. This control measure often gives mosquito control district the opportunity to stress ecologically sound methods while controlling mosquito. Toxorhynchites can be valuable management tool in areas where containers and tree hole contribute substantially to the standing crop of mosquitoes.

Introduction of Natural Enemy to the Model:



Figure(2): The new flow diagram

The natural enemy $S(t)$, is introduced to eat up the larva L (t) at the rate QUOTE $\alpha(t)$ $\alpha(t)$, then from equation (1) the equation L'(t) is deduced thus:

$$\frac{dL}{dt} = L'(t) = \sigma B(t) + \alpha(t)S(t) - \lambda L(t) - \mu L(t)$$

Where $$\alpha = \frac{c\beta S(t)}{L(t)}$$

$$\frac{dS}{dt} = S'(t) = b - \alpha S(t)$$

$$\lambda = 1/\alpha$$

Existence And Analysis Of The Free Equilibrium State

Here we would establish the general stability of the Model Free Equilibrium (MFE) state by considering the model parameters and using the model equations.

Since we have four systems of non-linear equations, we know that it is almost impossible to obtain an analytical solution of these systems. Therefore we use the idea of Beltrami conditions that;

If the determinants of the Jacobian matrix is greater than zero and the trace element of the Jacobian matrix is less than zero, then the equilibrium state of the model is locally and asymptotically stable.

4.3 The General Equilibrium States

The model equations are:

$$A'(t) = bN + \rho P(t) - \eta A(t) - \mu A(t) \quad \ldots (4.3.1)$$

$$B'(t) = \eta A(t) - (\sigma + \mu)B(t) \quad \ldots \quad \ldots \quad \ldots \quad (4.3.2)$$

$$L'(t) = \sigma B(t) + \alpha S(t) - (\lambda + \mu)L(t) \quad \ldots \quad \ldots \quad \ldots \quad (4.3.3)$$

$$P'(t) = \lambda L(t) - (\rho + \mu)P(t) \quad \ldots \quad \ldots \quad \ldots \quad (4.3.4)$$

$$J_E = \begin{bmatrix} -(\mu+\eta) & 0 & 0 & \rho \\ \eta & -(\sigma+\mu) & 0 & 0 \\ 0 & \sigma & -(\lambda+\mu) & 0 \\ 0 & 0 & \lambda & -(\rho+\mu) \end{bmatrix}$$

The contact matrix M is given by

$$M = \begin{bmatrix} -(\sigma+\mu) & 0 & 0 \\ \sigma & -(\lambda+\mu) & 0 \\ 0 & \lambda & -(\rho+\mu) \end{bmatrix}$$

$$-(\sigma+\mu+\lambda')\det(M) = [-(\lambda+\mu)\times(-(\rho+\mu))] - \lfloor 0\times\lambda \rfloor$$
$$= (\lambda+\mu)(\rho+\mu) - 0$$
$$= (\lambda+\mu)(\rho+\mu) > 0$$

Since all the parameters are positive then

$$\Rightarrow \det(M) > 0$$

Now the trace of the contact matrix is the sum of the diagonal of the contact matrix

$$\Rightarrow Trace(M) = [-(\sigma+\mu)] + [-(\lambda+\mu)] + [-(\rho+\mu)]$$

$$\Rightarrow Trace(M) = -[(\sigma+\mu) + (\lambda+\mu) + (\rho+\mu)] < 0$$

Since all the parameters are positive then it implies that $TraceM < 0$. Hence, the conditions of Beltrami are satisfied, therefore we can conclude that the equilibrium state is locally and asymptotically stable.

The Submodel With The Natural Enemy

$$\left.\begin{aligned} A'(t) &= bN + \rho P(t) - (\eta+\mu)A(t) \\ B'(t) &= \eta A(t) - (\sigma+\mu)B(t) \\ L'(t) &= \sigma B(t) + \alpha S(t) - (\lambda+\mu)L(t) \\ P'(t) &= \lambda L(t) - (\lambda+\mu)P(t) \\ S'(t) &= b - \alpha S(t) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots (4.4.1)$$

The Jacobian matrix associated with (4.4.1) is thus:

$$J_E = \begin{bmatrix} -(\mu+\eta) & 0 & 0 & \rho & 0 \\ \eta & -(\sigma+\mu) & 0 & 0 & 0 \\ 0 & \sigma & -(\lambda+\mu) & 0 & \alpha \\ 0 & 0 & \lambda & -(\rho+\mu) & 0 \\ 0 & 0 & 0 & 0 & -\alpha \end{bmatrix}$$

The contact matrix with natural enemy is

$$M = \begin{bmatrix} -(\lambda+\mu) & 0 & \alpha \\ \lambda & -(\rho+\mu) & 0 \\ 0 & 0 & -\alpha \end{bmatrix}$$

The $\det(M) = (\lambda+\mu)\det(M) = [-(\rho+\mu)\times(-\alpha)] - [0\times 0]$

$$= (\rho+\mu).\alpha - 0$$

$$\det(M) = \alpha(\rho+\mu) > 0$$

The $Trace(M) = [-(\lambda+\mu)] + [-(\rho+\mu)] + [-\alpha]$

$$Trace(M) = -[(\lambda+\mu) + (\rho+\mu) + \alpha] < 0$$

Result

Since the determinants of the Contact matrix is greater than zero and the trace of the contact matrix is less than zero, then we conclude that the equilibrium state of the model is locally and asymptotically stable.

The Equilibrium State Using R_O

It is a known fact from stability theorems that if R_O<1, it implies that the model

We re-categorize the population into two classes as follows:

$$N(t) = A(t), B(t), L(t), P(t), \alpha(t) S(t)$$

$X = [A(t), P(t)]$ and

$Z = [B(t), L(t), S(t)]$

Where

$$X = f(X, Z) = bN + \rho X - (\eta+\mu) X + \lambda Z - (\rho+\mu) X$$

$$= bN + \lambda Z - (\eta + 2\mu) X$$

$$Z = h(X, Z) = \eta X - (\sigma+\mu) Z + \sigma Z + \alpha Z - (\lambda+\mu) Z + b - \alpha Z$$

$$= \eta X - (\sigma+\mu) Z + (\sigma+\alpha) Z - (\lambda+\mu) Z + b - \alpha Z$$

Let $H = \frac{\delta h}{\delta z} = \sigma+\alpha-(\sigma+\mu)-(\lambda+\mu)-\alpha$

$$H = \sigma+\alpha-(2\mu+\sigma+\lambda+\alpha)$$

Let $H = M - D, M > 0, D > 0$, where D is a diagonal matrix then we have the basic reproduction number

$$R_0 = \phi(MD^{-1}) = \phi(M/D) = \frac{(\sigma+\alpha)}{(2\mu+\sigma+\lambda+\alpha)}$$

we have that,

$$R_O = \frac{(\sigma+\alpha)}{(2\mu+\sigma+\lambda+\alpha)} < 1$$

$$\Rightarrow R_O < 1$$

Conclusion:

We observe that based on the conditions of Beltrami which states, if the determinants of the Jacobian matrix is greater than zero and the trace is less than zero equations are locally asymptotically stable, otherwise unstable. Therefore, in this section we will establish the stability of free equilibrium states using the idea of Diekmann that is using reproduction number (R_O).

are satisfied then the equilibrium state is locally asymptotically stable and Diekmann criteria which says if $R_O < 1$ the Model Free Equilibrium (MFE) state is locally asymptotically stable.

From figure (2), we observed that to mathematical break the life cycle at larva stage, we need to control the number α. This means the more we introduce α into the cycle at the larva stage the fewer larvae that would survive to the next stage therefore, it is important to focus on α and choose a nice value for α so that we can have a smooth control programme. We also observed that λ is inversely proportional to α , this simply means if we increase the number α that would reduce the number of λ to a negligible value thereby cutting off the number of larvae leaving to the next stage at an epsilon value

From the analysis of the governing equations and based on the model assumptions, we saw that the stability of the free equilibrium states is locally asymptotically stable under those conditions mentioned. The derivation of the reproduction number using the idea of Diekmann also confirmed that the contact number R0 is strictly less than one which implies that the free equilibrium states of the model would be locally asymptotically stable. Therefore, from all the aforementioned We thereby conclude that when the natural enemy introduced is much, then the number of larvae moving to pupa will be almost zero and that will break the life cycle of the mosquito. Hence, this will reduce the spread of malaria in our society.

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